## Chapter 7

## Markets for Currency Swaps

As already discussed in Chapter 5, the choice of the currency of borrowing may be difficult; for instance, the currency that offers the lowest PV-ed risk spread may not be the most attractive one from the risk-management point of view. We also know how a firm can nevertheless have its cake and eat it: one can borrow in the lowcost denomination, and then swap that loan into the desired currency. The case we looked at was the simplest possible loan-one with just one single future payment, standing for interest and principal. In that case we $(i)$ convert the upfront inflow via a spot transaction from the currency of borrowing into the desired one, and (ii) convert the future outflow in the forward market, thus again replacing the currency of the loan's original outflow by the desired one.

But most loans with a life exceeding one year are, of course, multi-payment: interest is typically due at least once a year, and often even twice or four times; and also the principal can be amortized gradually rather than in one shot at the end. To swap such a loan, one would need as many forward hedges as there are future payments. The modern currency swap provides an answer to this: in one contract the two parties agree upon not just the spot conversion in one direction, but also the reverse conversions for all future service payments. The contract is typically set up such that the time pattern of the final payments corresponds to the time pattern of the original. For example, if the original loan is a bullet loan with a fixed interest rate, then the swapped package can also be of the bullet type and with a constant coupon. That last feature would not be achievable with a set of forward contracts: if the original loan has a constant coupon, then the converted coupons will vary depending on their due dates because the forward rates that we use for the conversion depend on the due date. With modern swaps we can even transform a currency-A floating-rate loan into a fixed-rate loan in currency B, something which cannot be done with simple forward contracts since the future service payments are not even known yet. So modern swaps are a general and flexible device to change one loan, chosen perhaps because of its low cost, into another loan that for some reason is viewed as more desirable. The second loan could be different from the original one in terms of currency, or interest payments (fixed versus floating), or
both. And these are just the plain-vanilla cases; many ad-hoc structures can be arranged at the customer's request.

This chapter is structured as follows. In the first section, we consider a landmark deal between two highly respected companies, the currency swap between IBM and the World Bank negotiated in 1981 (commonly viewed as the mother of the modern swaps) and we indicate the subsequent evolution of the swap into a standard, off-the-shelf product. We then show, in Section 7.2, how the modern currency swap works, and why such deals exist. An even more popular variant of currency swap is the interest-rate swap or fixed-for-floating rate swap, which we discuss in Section 7.3. Section 7.4, discusses a combination of the currency swap and the interest swap, called the fixed-for-floating currency swap or circus swap. Section 7.5 concludes this chapter.

### 7.1 How the Modern Swap came About

From Chapter 5 we know how spot-forward swaps can be used to transform one zero-coupon loan into a zero-coupon loan in a different currency. Swaps can also be used in themselves, as a package of back-to-back loans. The problem is that many of the applications are somewhat shady: shirking taxes, avoiding currency controls, not to mention laundering money. For this reason, back-to-back and parallel loans or spot-forward swaps were for a long time viewed as not $100 \%$ respectable. In 1981 all that changed. Two quite-above-board companies, IBM and the World bank, set up a contract which was quite clever and had a respectable economic purpose: avoiding transaction costs. There was a tax advantage too, but this was almost by accident.

The IBM-wB swap was a bilateral deal, very much tailor-made. But rapidly the swap became a standardised product offered routinely by banks. This evolution is depicted after our description of the IBM-wB deal.

### 7.1.1 The Grandfather Tailor-made Swap: IBm-wb

In 1981, IBM wanted to get rid of its outstanding DEM- and CHF-denominated callable debt because the USD had appreciated considerably and the DEM and CHF interest rates had also gone up. As a result of these two changes, the market value of ibm's foreign debt, expressed in terms of DEM and CHF, was below its face value, and the gap between market value and book value was even wider in terms of USD. IBM wanted to lock in this capital gain by replacing the DEM and CHF debt by new USD debt. However, in order to do this, IBM would have to incur many costs:

- IBM would have to buy DEM and CHF currency, thus incurring transaction costs in the spot market. In 1981 this was not yet the puny item it has become by now.
- Much more importantly, ibm's loans were callable indeed (that is, IBm could amortize them early) -but at a price above par. So IBM would have to fork out more than the DEM and CHF face value rather the economic value of the straight-bond component, which was below par. Calling would be like exercising an out-of-the-money option.
Finance theorists will happily point out that hedging the debt would be the obvious solution: borrow dollars and invest them in DEM and CHF assets that match the outstanding debt, thus neutralizing any possible re-appreciation of these currencies without any need to actually withdraw the old bonds. But CFO's will unhappily note that, in conventional accounting terms, this would double the debt.
- IBM would have to pay a capital-gains tax on the difference between the (dollar) book value and the price it paid to redeem the bonds.
- Lastly, ibm would have to issue new usd bonds to finance the redemption of its ChF and Dem debt. In those days, a bond issue costed at least a few percentages of the nominal value.

The World Bank (wb), on the other hand, wanted to borrow DEM and ChF to lend to its customers. Its charter indeed said that, currency by currency, its assets should be matched by its liabilities. Clearly, issuing new CHF and DEM bonds would have entailed issuing costs.

To sum up, IBM wants to withdraw CHF and DEM bonds (at a rather high cost) while wB wants to issue ChF and dem bonds (also at a cost) (see Figure 7.1). To avoid all of these expenses, IBM and wB agreed that

- the wB would not borrow CHF and DEm, but would borrow USD instead. With the proceeds, it would buy spot CHF and DEM needed to make loans to its customers.
- the wB undertook to take over the servicing of IBM's outstanding DEM and CHF loans, while IBM promised to service the wB's (new) usd loan.

This way, each party achieved its objective. IBM has effectively traded (or swapped) its DEM and CHF obligations for USD obligations: its DEM-CHF debt is taken care of by wB, economically, and IBM now services USD bonds. The WB, on the other hand, has an obligation to deliver DEM and CHF, which is what wB needed. One obvious joint saving of the swap was the cost of issuing new WB bonds in DEM and CHF, and redeeming the old IBM loans in DEM and CHF. Also, the recognition of IBM's capital gain was postponed because the old bonds were not redeemed early. Another saving was that the WB could issue USD bonds at a lower risk spread than IBM. ${ }^{1}$

[^0]Figure 7.1: The IBM-WB swap


Key Top left: the initial situation; top-right: the originally intended final situation; bottom: how the essence of the desired solution was realized, at a lower cost.

Of course, the amounts to be exchanged had to be acceptable to both parties. The present value of IBM's USD payments to the wB should, therefore, be equal to the present value of the DEM and CHF inflows received from the wb.

## Example 7.1

Assume, for simplicity, that IBM has an outstanding DEM debt with a face value of DEM 100 m and a book value of USD 60 m (based on the historic USD/DEM rate of 0.6 ), maturing after five years and carrying a 5 percent annual coupon. Assume the current five-year DEM interest rate is 10 percent and the DEM now trades at USD/DEM 0.4. In DEM, IBM's existing debt would have a present value of ${ }^{2}$

$$
\begin{equation*}
\text { DEM } 100 \mathrm{~m} \times[1+(0.05-0.1) \times a(10 \%, 5 \text { years })]=\text { DEM } 81.05 m, \tag{7.1}
\end{equation*}
$$

where $a(r, n)$ is the present value of an n-year unit annuity discounted at a rate $r$ :

$$
\begin{equation*}
a(r, n) \stackrel{\text { def }}{=} \sum_{t=1}^{n} \frac{1}{(1+r)^{t}}=\frac{1-(1+r)^{-n}}{r} . \tag{7.2}
\end{equation*}
$$

At the current spot rate of USD/DEM 0.4 , WB's undertaking to service this debt is worth $81.05 \times 0.4=$ USD 32.42 m .

[^1]The equal-value principle requires that ibm's undertaking have the same present value. Thus, the USD loan (issued at the then-prevailing rate for five years) must have a present value of USD 32.42 m .

As we have argued, one purpose of the entire IBM/wB deal was to avoid transaction costs. A nice by-product, in terms of taxes, was that IBM locked in its capital gain on its foreign currency debt without immediately realizing the profit. Let us quantify some of these elements using the above figures. If ibm had called its DEm debt at 102 percent of its DEM par value, the cost of withdrawing the debt would have been $100 \mathrm{~m} \times 1.02 \times$ USD $/$ DEM $0.4=$ USD 40.8 m , thus realizing a taxable capital gain of USD $60 \mathrm{~m}-40.8=$ USD 19.2 m . In contrast, under the swap, the DEM debt remains in IBM's books for another five years. That is, in accounting terms, the capital gain will be realized only when, five years later, IBM pays the swap principal (USD 32.42 m ) to the WB and receives DEM 100 m to redeem its DEM debt. In short, the swap also allowed IBM to defer its capital gains taxes.

### 7.1.2 Subsequent Evolution of the Swap Market

We know that a forward contract is like an exchange of two initially equivalent promissory notes, one in HC and one in FC. In the IBM-wB deal we see, instead, something like an exchange of two bonds (or a least cash-flow patterns that correspond to bond servicing schedules). This differs from the forward contract in the sense that there is not just an exchange of two main amounts at the end, but also interim interest is being paid to each other at regular dates. But the principle of initial equivalence of the two "legs" of the deal is maintained.

One feature that has changed nowadays, relative to the IBM-WB example, is that almost invariably a reverse spot exchange is added. One reason is that very often the purpose of the swap is to transform a loan taken up in currency X into one expressed in currency Y; and to do that, one also needs the immediate currency- Y inflow beside the future outflows.

## DoItYourself problem 7.1

Suppose you want to borrow GBP, but what you actually do is borrow USD and swap, the way we saw it in Chapter 5. So part of the deal is that you promise the swap dealer a stream of GBP; the swap dealer in return then pays you USD with which you can service your bank loan. But all this only delivers you the future-GBP-outflow part of the desired loan. To get also the immediate-GBP-inflow part, you convert the USD proceeds of the bank loan into pounds.

A second reason for adding the spot deal is that the exchange of time- $t$ PVs simplifies the negotiation process. One has to realize, indeed, that swaps are typically add-ons to biggish loans; and taking up a big loan is a much rarer and slower decision than, say, a spot or forward transaction that has to do with trade transactions. Since negotiations take hours or days, and since the spot rate is moving all the time, one
would have to continuously change one leg of the swap to maintain initial equivalence of the future payments. By throwing in an exchange of the spot PVs, this problem is much reduced. The idea is that one can still get zero initial value for the swap as a whole if the Net PV of each leg separately is zero-the PV of the future payments minus the initial flow in the opposite direction.

## Example 7.2

Suppose that, at the beginning of the negotiations, a uS company promises to send to a Dutch company a stream of USD corresponding to a bullet loan with notional value USD 50 m at 4 percent payable annually. Suppose the normal yield rate for this type of bond is 4 percent, so that $\mathrm{PV}_{\text {USD }}=$ USD 50 m . Company B promises a stream of EUR in return. On the basis of $S_{t}=$ USD/EUR 1.25 and a EUR interest rate of 4.5 percent, the EUR payments would mirror the service payments for a EUR 40 m loan at 4.5 percent. This way, the PVS of the EUR and USD streams are identical, resulting in a zero total value of the contract.

But if one hour later the spot rate is 1.26 , the calculations would have to be revised. This revision would become unnecessary if the contract also stipulates an initial exchange of EUR 40 m for USD 50 m . Then, to the US company, the appreciation of the EUR increases the USD PV of the incoming future Euros but also increases by the same factor the USD value of the EUR amount the company needs to fork out immediately. Thus, the net value of the Eur leg remains zero as long as interest rates do not change.

With the immediate exchange of principals brought in, one can do with approximate equivalence of the two notional amounts. An approximate equivalence is still important because the two loans also serve as security for each other. If one side were far smaller than the other, the security provision would be unacceptably asymmetric.

A second major change, relative to the IBM-WB example is that contracts are now standardized. The early swaps were carefully negotiated between two parties, with task forces of financial economists and lawyers in attendance to calculate the gains and to arrive at a fair division of the gains. Inevitably, then, one huge initial problem was to find a counterpart with the complementary objectives. In forward markets, we know, banks act as intermediaries. If company A buys forward, the bank agrees, and afterwards solicits a sale from someone else by skewing its bids (Chapter 3), or the bank closes out in the spot and money markets (synthetic sale). This is exactly how things have become in the swap markets too. A company signs a swap agreement with a bank, which may keep this contract "on its book" (i.e. open) for a while, until new contracts have brought the overall book closer to neutrality. If the risk is too large, the bank can always hedge in the bond and spot markets (synthetic swap). This hedging was easiest in the USD interest-rate swap market, where the two notional loans that constitute the swap are expressed in the same currency but have different interest forms - typically, one leg fixed-rate and the other floatingrate. Given that there is a huge market for similar fixed- and floating-rate bonds
outstanding, swap dealers could easily close out in the bond market. Also, a lively secondary market for swaps has emerged.

We are now ready to have a closer look at the how swaps are set up. We begin with "fixed-for-fixed" currency swaps, that is, swaps with fixed coupons in each leg.

### 7.2 The Fixed-for-Fixed Currency Swaps

### 7.2.1 Motivations for Undertaking a Currency Swap

The reasons for using swaps are essentially those mentioned for spot-forwards swaps (Chapter 5). Generally, the point is to avoid unnecessary costs generated by market imperfections, primarily information costs that lead to excessive risk spreads asked by uninformed banks. The IBM-wB case was mainly a transaction-cost motivated structure. Also the advantages of off-balance-sheet reporting remain valid, at least when a swap is compared to its synthetic version (borrow in one currency and invest in another).

An extreme form of market imperfection arose in one particular instance: in the early 1980s, the French car manufacturer Renault wanted to borrow Yen and use the proceeds to redeem outstanding USD debt, but found that in those days the Yen bond market was quasi closed to foreign borrowers. So Renault swapped its USD loan with Yamaichi Securities for JPY debt. The Renault-Yamaichi swap was not a fixed-for-fixed swap, so its discussion is deferred to Section 7.4, below.

### 7.2.2 Characteristics of the Modern Currency Swap

In many ways, the modern fixed-for-fixed currency swap is simply a long-term version of the classical spot-forward swap. A fixed-for-fixed currency swap can be defined as a transaction where two parties exchange, at the time of the contract's initiation, two principals denominated in different currencies but with (roughly) the same market value, and return these principals to each other when the contract expires. In addition, they periodically pay a normal interest to each other on the amounts borrowed. The deal is structured as a single contract, with a right of offset. The features of a fixed-for-fixed currency swap are described in more detail below.

## Swap rates

In a fixed-for-fixed currency swap, the interest payments for each currency are based on the currency's "swap (interest) rate" for the swap's maturity. These swap rates
are simply yields at par for near-riskless bonds with the same maturity as the swap. ${ }^{3}$ In practice, the swap rates are close to the long-term offshore rates on high-quality sovereign loans, that is, loans by governments. For the following reasons, it is appropriate to use near-risk-free rates to compute the interest on the amounts swapped even if the counterpart in the contract is not an AAA company:

- The bank's risks in case of default are limited because of the right-of-offset clause. In unusually risky cases, the contract parties also have to post margin.
- The probability of default is small. This is because the customers are screened; small or low-grade companies get no chance, or have to post initial margin.
- In addition, many swap contracts have a "credit trigger" clause, stating that, if the customer's credit rating is revised downward, the financial institution can terminate the swap, and settle for the swap's market value at that moment. Thus, the bank has an opportunity to terminate the contract long before default actually occurs - unless the company goes straight from AA to failure, Enron-style.
- Finally, because of the right of offset, the uncertainty about the bank's inflows is the same as the uncertainty about the bank's outflows. The fact that the uncertainties are the same implies that the corrections for risk virtually cancel out. That is, it hardly matters whether or not one adds a similar (and small) default risk premium to the risk-free rates when one discounts the two cash flow streams. The effect of adding a small risk premium when valuing one "leg" of the swap will essentially cancel out against the effect of adding a similar risk premium in the valuation of the other leg.

Look at the rates in Figure 7.2. Sterling has a one-year swap rate of 4.96-4.99. Elsewhere in the same FT issue I find the following 1-year rates: Interbank Sterling 4.875-4.96875, BBA Sterling 4.65625, Sterling CD 4.90625-4.9375, Local authority depts 4.875-4.9375. Thus, the swap rate is close to a risk-free rate. There is a small risk premium, but it is so low that for all practical purposes you can think of the swap rate as the risk-free rate, the same way LIBOR is called risk-free.

The Key to the fT table mentions another detail: a swap rate is quoted against a particular floating rate. This is from the fact that the busiest section of the swap market is the interest swap, fixed versus floating or vice versa. In principle it should not matter what exactly the floating-rate part is: since investors can freely

[^2]Figure 7.2: Swap Rates as quoted in the Financial Times

|  | Euro-C |  | £ Stlg |  | SwFr |  | US \$ |  | Yen |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 06/06/06 | bid | ask | bid | ask | bid | ask | bid | ask | bid | ask |
| 1 year | 3.42 | 3.45 | 4.96 | 4.99 | 1.91 | 1.97 | 5.46 | 5.49 | 0.60 | 0.62 |
| 2 year | 3.64 | 3.67 | 5.03 | 5.07 | 2.26 | 2.34 | 5.41 | 5.43 | 0.92 | 0.95 |
| 3 year | 3.77 | 3.80 | 5.08 | 5.12 | 2.46 | 2.54 | 5.41 | 5.43 | 1.18 | 1.21 |
| 4 year | 3.86 | 3.89 | 5.09 | 5.14 | 2.59 | 2.67 | 5.42 | 5.45 | 1.39 | 1.42 |
| 5 year | 3.93 | 3.96 | 5.09 | 5.14 | 2.69 | 2.77 | 5.45 | 5.47 | 1.56 | 1.60 |
| 6 year | 3.99 | 4.02 | 5.09 | 5.14 | 2.78 | 2.86 | 5.46 | 5.50 | 1.71 | 1.74 |
| 7 year | 4.05 | 4.08 | 5.08 | 5.12 | 2.85 | 2.93 | 5.49 | 5.51 | 1.84 | 1.87 |
| 8 year | 4.10 | 4.13 | 5.06 | 5.11 | 2.91 | 2.99 | 5.51 | 5.54 | 1.95 | 1.98 |
| 9 year | 4.15 | 4.18 | 5.04 | 5.09 | 2.97 | 3.05 | 5.53 | 5.56 | 2.04 | 2.06 |
| 10 year | 4.20 | 4.23 | 5.01 | 5.07 | 3.02 | 3.10 | 5.55 | 5.58 | 2.11 | 2.14 |
| 12 year | 4.28 | 4.31 | 4.96 | 5.03 | 3.08 | 3.18 | 5.58 | 5.62 | 2.24 | 2.27 |
| 15 year | 4.38 | 4.41 | 4.88 | 4.97 | 3.17 | 3.27 | 5.63 | 5.66 | 2.38 | 2.41 |
| 20 year | 4.47 | 4.50 | 4.75 | 4.88 | 3.26 | 3.36 | 5.66 | 5.69 | 2.54 | 2.57 |
| 25 year | 4.51 | 4.54 | 4.64 | 4.77 | 3.27 | 3.37 | 5.66 | 5.70 | 2.63 | 2.66 |
| 30 year | 4.52 | 4.55 | 4.56 | 4.69 | 3.26 | 3.36 | 5.66 | 5.69 | 2.67 | 2.70 |

Bid and ask rates are as of close of London business. US $\$$ is quoted annual money actual/360 basis against 3 months Libor, pound and Yen quoted on a semi-annual actual/365 basis against 6-months Libor, Euro/Swiss Franc rate quoted on annual bond $30 / 360$ basis against 6 -month Euribor Libor with the exception of the 1 year rate which is quoted against 3 month Euribor/Libor. Source: ICAP plc
chose between say 3 - and 6 - or 9 -month LIBOR, the three should be equivalent. In practice, differences in e.g. liquidity may cause the swap rate to differ, in a minor way, depending on what the floating-rate part is.

## Costs

The swapping bank charges a small annual commission of, say, USD 200 on a USD 1 m swap, for each payment to be made. Most often this fee is built into the interest rates, which would raise or lower the quoted rate by a few basis points.

## Example 7.3

Suppose that the seven-year yields at par are 3.17 percent on USD and 3.9 percent on Eur. The swap dealer quotes

$$
\begin{aligned}
& \text { USD } 3.15 \%-3.19 \% \text {, } \\
& \text { EUR } 3.88 \%-3.92 \% \text {. }
\end{aligned}
$$

If your swap contract is one where you "borrow" EUR and "lend" USD, you would then pay 3.92 percent on the EUR, and receive 3.15 percent on the USD.

Theoretically, the series of future commissions, one per payment, might be replaced by a single up-front fee with a comparable present value. Even if this is seldom done in practice, it is still useful for you to always calculate this number, so as to have an idea of the overall cost. For a ten-year USD 1m swap at 3 percent annually that has a USD 200 commission per payment, the equivalent up-front commission would be about $200 \times \mathrm{a}(3 \%, 10$ years $)=200 \times 8.530,203=$ USD 1706 , or

Table 7.1: Fixed-for-fixed Currency Swap: the Interim Solution

|  | loan | Swap |  | Combined |
| :--- | :---: | :---: | :---: | :---: |
|  | JPY 1000 borr'd <br> at $1 \%$ | JPY 1000 lent, <br> at $0.6 \%$ | USD 10 m borr'd <br> at $3 \%$ |  |
| principal at $t$ | JPY 1000 m | <JPY $1000 \mathrm{~m}>$ | USD 10 m | USD 10 m |
| interest (p.a.) | $<$ JPY $10 \mathrm{~m}>$ | JPY 6 m | <USD $0.3 \mathrm{~m}>$ | <JPY $4 \mathrm{~m}>\&$ |
| <USD $0.3 \mathrm{~m}>$ |  |  |  |  |
| principal at $T$ | $<$ JPY $1000 \mathrm{~m}>$ | JPY 1000 m | <USD $10 \mathrm{~m}>$ | <USD $10 \mathrm{~m}>$ |

0.17 percent of the face value.

Thus, although the swap remains a zero-value contract, the customer has to pay a small commission. (You can tell the difference between a price and a commission because the commission is always paid, whether one goes long or short; in contrast, the price is paid if one buys, and is received if one sells.) The commissions in the swap are small because the costs of bonding and monitoring are low-default risk is minimal anyway, as we saw-and because the amounts are large. (A typical interbank swap transaction is for a few million USD, and the Reuters swap-dealing network requires minimally USD 10 m ; for corporations, swaps can be smaller but contracts below USD 1 m are rare.) Familiarly, the swap spread also depends on liquidity. Deep markets like USD, EUR and JPY, in Figure 7.2, have spreads of 3 bp or thereabouts, but for CHF and GBP the margin is wider, rising to 10 bp at the far end of the maturity spectrum.

## How to Handle and Compare Risk Spreads

Suppose a Japanese company wants to borrow cheaply in JPY ( $=\mathrm{HC}$ ) from its house bank, at 1 percent for 7 years, bullet, and then swap the loan into USD. The swap rates quoted are 0.6 percent on JPY and 3 percent on USD. In Table 7.2 I set up a little tabular that shows you, in the first column of figures, the original loan (JPY at $1 \%$ ); in the next two, the twin legs of the swap; and, lastly, the combined cash flow (loan and swap). The version I show in that table is, actually, rarely applied in practice; I mainly use it as an interim step because it helps to explain the advantage of the swap as well as the logic of the ultimate solution. The spot rate being about JPY/USD 100 , we work with notional principals of JPY 1000 m and USD 10 m . So the company

- borrows JPY 1000 m from the house bank at 1 percent (the actual loan rate),
- "re-lends" these JPY 1000 m to the swap dealer, at $0.6 \%$ (the JPY swap rate),
- ... who in return "lends" USD 10 m to the firm at $3 \%$ (the USD swap rate).

This is summarized in Table 7.1. Note how the company borrows, ultimately,

## Table 7.2: Fixed-for-fixed Currency Swap: Marked-up uSD rate

|  | $\begin{gathered} \text { loan } \\ \text { JPY } 1000 \text { borr'd } \\ \text { at } 1 \% \end{gathered}$ | swap |  | Combined |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} \text { JPY } 1000 \text { lent, } \\ \text { at } 1 \% \end{gathered}$ | USD 10 m borr'd at $3.438823 \%$ |  |
| principal at $t$ <br> interest (p.a.) <br> principal at $T$ | $\begin{gathered} \text { JPY } 1000 \mathrm{~m} \\ <\text { JPY } 10 \mathrm{~m}> \\ <\text { JPY } 1000 \mathrm{~m}> \end{gathered}$ | $\begin{gathered} <\text { JPY } 1000 \mathrm{~m}> \\ \text { JPY } 10 \mathrm{~m} \\ \text { JPY } 1000 \mathrm{~m} \end{gathered}$ | $\begin{gathered} \text { USD } 10 \mathrm{~m} \\ <\text { USD } 343,882.30> \\ \text { <USD } 10 \mathrm{~m}> \end{gathered}$ | $\begin{gathered} \text { USD } 10 \mathrm{~m} \\ <\text { USD } 343,882.30> \\ \text { <USD } 10 \mathrm{~m}> \end{gathered}$ |

USD 10 m , with an annual interest payment consisting of the USD risk-free rate (3 percent) plus a risk spread which is, very literally, the risk spread on a JPY loan from the house bank: $1 \%-0.6 \%=0.4 \%$ on JPY $1000 \mathrm{~m} .{ }^{4}$

The above solution is still somewhat unelegant because the company pays part of its annual interest payments in JPY, an undesirable feature if it basically wants a usD loan. There are two simple solutions:

- either replace the seven annual JPY 4 m payments by an equivalent upfront fee, which is of course their PV:

$$
\begin{align*}
\text { Equivalent upfront fee } & =4 m \times a(0.6 \%, 7 \text { years }) \\
& =4 m \times 6.834,979=\mathrm{JPY} 27.339,917 m, \tag{7.3}
\end{align*}
$$

- or replace it by an equivalent USD annuity:

$$
\text { Find } \begin{align*}
X^{*} \text { such that } & \underbrace{S_{t} \times \overbrace{X^{*} \times a(3 \%, 7 \text { years }}^{\text {PV of annuity usD } X^{*}}}_{\ldots \text { translated into JPY }}=\underbrace{4 m \times a(0.6 \%, 7 \text { years })}_{\text {PV of annuity JPY } 4 \mathrm{~m}} \\
\Rightarrow X^{*} & =\frac{4 m}{S_{t}} \frac{a(0.6 \%, 7 \text { years })}{a(3 \%, 7 \text { years })} \\
& =\frac{4 m}{100} \frac{6.834,979}{6.230,282}=\operatorname{USD} 43,882.30 \mathrm{~m} \tag{7.4}
\end{align*}
$$

Technically, we ask the swap dealer to pay us $1 \%$ (our borrowing rate) on the yens instead of $0.6 \%$ (the swap rate), and in return we increase the dollar interest paid to the swap dealer by the equivalent amount. Table 7.2 summarizes the modified solution.

The second solution immediately allows us to discover whether the swapped loan is more attractive than a direct USD loan (an alternative we have not yet looked

[^3]at). The translated risk spread equivalent to the $0.4 \%$ charged by the Japanese housebank, as a percentage of the USD 10 m borrowed, amounts to $43,882.30 / 10 \mathrm{~m}=$ $0.438823 \%$. Let's denote the risk spreads by $\rho$ and $\rho^{*}$, as in Chapter 5, and let's use $s$ and $s^{*}$ to refer to the swap rates. You can check that the generalized equivalence condition is
\[

$$
\begin{equation*}
\rho^{*} \stackrel{\text { equiv }}{=} \rho \frac{a(s, n)}{a\left(s^{*}, n\right)} \Leftrightarrow \underbrace{\rho^{*} \times a\left(s^{*}, n\right)}_{\mathrm{PV} \text { of FC risk spread }} \stackrel{\text { equiv }}{=} \underbrace{\rho \times a(s, n)}_{\mathrm{PV} \text { of HC risk spread }} \tag{7.5}
\end{equation*}
$$

\]

Thus, a borrower gains from the swap if the spread quoted for a direct loan is higher than this translated HC risk spread, the HC figure projected into a different interestrate environment via the adjustment $\times a(s, n) / a\left(s^{*}, n\right)$. Similarly, a credit analyst working for a bank can use the formula to consistently translate the borrower's HC risk spread into FC. The solution is a straightforward generalization of the one for simple spot-forward swaps in Chapter 5: a FC risk spread $\rho^{*}$ is equivalent to a HC $\rho$ if their PVs are the same. The only change is that, of course, the PV'ing now involves annuities rather than a single payment: in a bullet loan, the risk premium is paid many times, not just once. Note also that for non-bullet loans the above formula no longer works, because the risk-spread payments (in amounts, not percentages) then no longer are constant. The equal-PV rule for equivalence still would hold, but the computations would be messier.

Also the intuition as to why and when a translated risk spread exceeds the original one remains the same as before. In risk-adjusted terms, the Yen is the strong currency here, as we can infer from its lower interest rate. So a strip of 0.4 percent payments in USD cannot be as good as a series of 0.4 percent in Yen, the strong currency. The above formula tells us exactly how the strength of the currencies, as embodied in their interest rates, has to be quantified in the translation process: taking into account the relative annuity factors, one needs to offer $0.438823 \%$ in USD to be in balance with $0.4 \%$ in Yen.

## Non-bullet loans

Standard swap-rate quotes are for bullet loans. Any other package is replicated as a combination of bullet loans with different times to maturity, and for each component the appropriate swap rate holds.

## Example 7.4

Assume the swap rates for 1,2 , and 3 years are 5,6 , and $7 \%$, respectively. We want to create a three-year constant-annuity loan, with three payments worth 1000 each. The tools we have are three bullet loans: a one-year specimen with face value $V_{1}$ (to be determined); a two-year one with face value $V_{2}$; and a three-year loan with face value $V_{3}$.
Finding the replication requires solving a simple linear system. In the case of a

Table 7.3: Replicating a constant-annuity loan from bullet loans

|  | interest payments on |  |  | amortization payments on <br>  <br>  <br>  <br>  <br> loan maturing in year ... <br> loan maturing in year ... <br>  <br>  <br> 1 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 1 | 2 | 3 |  |  |
| year 1 | 41.985 | 52.901 | 65.421 | 839.694 | 0 | 0 | 1000 |
| year 2 | 0 | 52.901 | 65.421 | 0 | 881.679 | 0 | 1000 |
| year 3 | 0 | 0 | 65.421 | 0 | 0 | 934.579 | 1000 |

Key The loans are 839.694 for one year, 881.679 for two years, and 934.579 for three - just believe me, or read Figure 7.3. The annual interest payments are $5 \%$ (one year loan), $6 \%$ (two) or $7 \%$ (three), and each loan is amortized on the promised dates. The total combined service schedule is exactly 1000 every year.

Figure 7.3: Replicating a constant-annuity loan from bullet loans

$$
\begin{aligned}
& \begin{array}{|c||c||c|}
\hline \mathrm{V} 1 & & \\
& \mathrm{~V} 2 & \\
\mathrm{~V} 2 & \mathrm{~V} 3 \\
\mathrm{C} 1 & & \\
\hline \mathrm{C} 2 & \mathrm{C} 2 & \\
\hline \hline \mathrm{C} 3 & \mathrm{C} 3 & \mathrm{C} 3 \\
\hline
\end{array} \\
& \mathrm{~V}_{3}+\mathrm{C}_{3}=1000 \\
& \mathbf{V}_{3}\left(1+\mathrm{s}_{3}\right)=1000 \Rightarrow \mathrm{~V}_{3} \\
& \begin{aligned}
\mathrm{V}_{2}+\mathrm{C}_{2}+\mathrm{C}_{3}=1000 \\
\mathbf{V}_{2}\left(1+\mathrm{s}_{2}\right)+\mathrm{V}_{3} \mathrm{~s}_{3}=1000 \Rightarrow \mathrm{~V}_{2}
\end{aligned} \\
& \begin{aligned}
\mathrm{V}_{1}+\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3}=1000 \\
\mathbf{V}_{1}\left(1+\mathrm{s}_{1}\right)+\mathrm{V}_{2} \mathrm{~s}_{2}+\mathrm{V}_{3} \mathrm{~s}_{3}=1000 \Rightarrow \mathrm{~V}_{1}
\end{aligned} \\
& \mathrm{~V}_{3}=934.58 \mathrm{~V} 2=881.68, \mathrm{~V} 1=839.69
\end{aligned}
$$

Key A service schedule amounting to three times 1000 is arranged as follows:

- We begin with year 3. In that year, only the three-year loan is still alive, and its total service cost including $7 \%$ interest must be 1000 . So the requirement is to find $V_{3}$ such that $V_{3} \times 1.07=1000-$ that is, $V_{3}=1000 / 1.07=934.579$. The balance is interest on the 3 -year bullet loan.
- Of the total 1000 paid in year 2 , the same 934.579 is available, after paying the interest in the three-year bullet loan, for principal and coupon of the two-year bullet loan: $V_{3}=V_{2} \times 1.06$. So $V_{2}=V_{3} / 1.06=881.678$, etc.
constant-annuity loan the rule is that $V_{t}=V_{t+1} /\left(1+s_{t}\right)$, with a "dummy" $V_{4}$ defined as the annuity itself-that is, $V_{3}=1000 / 1.07=934.579, V_{2}=934.579 / 1.06=$ 881.678, and $V_{1}=881.679 / 1.05=839.694$. Table 7.3 verifies that this indeed produces a combined cash flow for the three loans together of 1000 every year, and Figure 7.3 show you how you get these numbers.

This way, the swap dealer has also found that the PV of the three-year annuity is $934.58+881.68+839.69=2,655.95$. Spreadsheet afficionados will readily confirm that this corresponds to an IRR of $6.347 \%$. This would then be the swap dealer's rate for three-year constant-annuity loans. As you see, this is neither the 3 -year rate nor the 2- or 1-year rate for bullet loans, but a complicated mixture of all three. But this is the swap dealer's problem: the user can just work with the swap rate
given to her for the particular type of loan at hand.

## Valuing an Outstanding Fixed-for-Fixed Currency Swap

The last issue that we discuss in the section on fixed-for-fixed currency swaps is the valuation of such a swap after its inception. An assessment of the market value of a swap is required for the purpose of true and fair reporting to shareholders and overseeing authorities, or when the contract is terminated prematurely (by negotiation, or by default, or by the credit trigger clause).

Just as a forward contract, the fixed-for-fixed currency swap acquires a non-zero value as soon as the interest rates change, or as the spot rate changes. Since a swap is like a portfolio of (a) a loan and (b) an investment in long-term deposits (or in bonds), we can always value a swap as the difference between the market value of the loan and the market value of the investment.

## Example 7.5

Two years ago, a bank swapped a company loan (asset) of USD 100 m for GBP 50 m for seven years, at the swap rates of 4 percent on the USD leg and 5 percent on the GBP leg. This reflected the long-term interest rates and the spot rate of USD/GBP 2 prevailing when the contract was signed. Now the five-year USD swap rate is 2.5 percent, the five-year GBP swap rate is 4 percent, and the spot rate is USD/GBP 1.7. The procedure suggested by the International Swap Dealers Association is to value the swap by applying the traditional bond valuation formula to each of the swap's legs. Thus, the company's USD outflows are valued as

$$
P V_{\text {USD }}=100 m \times[1+(0.04-0.025) \times a(2.5 \%, 5 \text { years })]=\text { USD } 106,968,742.74,(7.6)
$$

while its GBP inflows are worth

$$
\begin{equation*}
\mathrm{PV}_{\mathrm{GBP}}=50 m \times[1+(0.05-0.04) \times a(4 \%, \text { 5years })]=\operatorname{GBP} 52,225,911.17 \tag{7.7}
\end{equation*}
$$

At the spot rate of USD/GBP 1.7, these GBP inflows are worth USD 88,784,048.98. The contract has therefore become a net liability, with value USD 88,784,048.98-$106,968,742.74=-$ USD $18,184,693.76$.

This finishes our discussion of the fixed-for-fixed currency swap. We now turn to other types of swaps, the most important of which is the interest rate swap or coupon swap.

### 7.3 Interest Rate Swaps

In an interest rate swap, there is still an exchange of the service payments on two distinct loans. However, the two loans involved now differ not by currency, but
by the method used to determine the interest payment (for instance, floating rate versus fixed rate). Because both underlying loans are in the same currency, there is no initial exchange of principals and no final amortization. In that sense, the two loans are notional (fictitious, or theoretical). The only cash flows that are swapped are the interest streams on each of the notional loans. In short, parties A and B simply agree to pay/receive the difference between two interest streams on the notional loan amounts.

The standard interest swap is the fixed-for-floating swap or coupon swap. The base swap is rarer. We discuss each of them in turn.

### 7.3.1 Coupon Swaps (Fixed-for-Floating)

We now describe the characteristics of a fixed-for-floating swap and how one can value such a financial contract.

## Characteristics of the Fixed-for-Floating Swap

In our discussion of the fixed-for-fixed currency swap, we saw that, in terms of the risk spread above the risk-free rate, a firm often has a comparative advantage in one currency but may prefer to borrow in another currency. The firm can retain its favorable risk spread and still change the loan's currency of denomination by borrowing in the most favorable market and swapping the loan into the preferred currency. The same holds for the fixed-for-floating swap except that, instead of a preferred currency, the firm now has a preferred type of interest payment. For instance, the firm may have a preference for financing at a fixed rate, but the risk spread in the floating-rate market may be lower. To retain its advantage of a lower spread in the floating-rate market, the firm can borrow at a floating rate, and swap the loan into a fixed-rate loan using a fixed-for-floating swap.

Because the swap contract is almost risk free, the interest rates used in the swap contract are (near) risk-free rates. For the floating-rate leg of the swap, the rate is traditionally LIBOR or a similar money market rate, while the relevant interest rate for the fixed-rate leg is the same N -year swap rate as used in fixed-for-fixed currency swaps. In fact, traditionally, the fixed swap rate was defined as the rate which the swap dealer thought to be as good as LIBOR, that is, which she or he was willing to take as the fixed-rate leg in a fixed-for-floating or floating-for-fixed swap. Also, LIBOR in currency X is also defined as acceptable against LIBOR in currency Y , which in turn must be acceptable against currency-Y fixed.

## Example 7.6

An AA Irish company wants to borrow NZD to finance (and partially hedge) its direct investment in New Zealand. Because the company is better known in London than in Auckland, it decides to tap the euro-NZD market rather than the loan market in New Zealand. As NZD interest rates are rather volatile, the company prefers fixed-rate

Table 7.4: Fixed-for-floating swap

|  | loan |  | Swap |  |
| :--- | :---: | :---: | :---: | :---: |
|  | NZD 1 borr'd | NZD 1 lent, | NZD 1m borr'd |  |
|  | at LIBOR $+1 \%$ | at LIBOR $\%$ | at 5\% |  |$]$

loans. But eurobanks, which are funded on a very short-term basis, dislike fixedrate loans, which means that the company would have to tap the bond market. The company's alternatives are the following:

- A euro-NZD fixed-rate bond issue would be possible only at 7 percent, which represents a hefty 2 percent spread above the NZD swap rate of 5 percent.
- From a London bank, the Irish company can get a NZD floating-rate bank loan at LIBOR +1 percent.

The company can keep the lower spread required in the floating-rate market and still pay a fixed rate, by borrowing NZD at the NZD LIBOR +1 percent, and swapping this into a fixed-rate NZD loan at the 5 percent swap rate. The payment streams, per NZD, are summarized in Table 7.4. To help you see the link between the payments under the swap contract and the underlying notional loans, we have added the theoretical principals at initiation and at maturity. In practice, the principals will not be exchanged. We see that this company borrows foreign currency at the NZD risk-free fixed rate ( 5 percent) plus the spread of 1 percent it can obtain in the "best" market (the floating-rate eurobank-loan market). Therefore, the company pays 6 percent fixed rather than the 7 percent that would have been required in the bond market.

Having done the number-crunching, let's talk economics now: how it is possible that the bond market requires 2 percent, by way of risk spread, when banks are happy with 1 percent? One reason is that banks are quite good at credit analysis, while Swiss dentists - still a non-trivial part of the bond-market clientèle - are not trained analysts. Also, the amounts at stake for a bank do justify a thorough analysis, while the 10,000 dollars invested by the Swiss dentists are too small for this. Furthermore, our Irish company will be happy to privately provide information to its bank that it would not dream of publishing in a prospectus. In short, the bank knows more, and knows better what the information means.

The swap dealer, who has to find a new party with (roughly) the opposite wants as our AA company, might then talk to an institutional investor, like an insurance company. They like long-duration deals. So everybody is happy. The insurance company gets a long-run fixed-rate investment and the firm the long-run fixed-rate
funding, but the credit analysis and the first-line default risk are left to the credit specialist, the bank.

From the above discussion, it is obvious that the potential advantages of the coupon swap are similar to the ones mentioned in the case of the fixed-for-fixed currency swap. What remains to be discussed is how to determine the value of the fixed-for-floating swap.

## Valuing a Fixed-for-Floating Swap

We have seen that in a fixed-for-floating swap without default risk, the incoming stream is the service schedule of a risk-free floating-rate loan, and the outgoing stream is the service schedule of a traditional risk-free fixed-rate loan (and vice versa for the other contract party). The fixed-rate payment stream is easily valued by discounting the known cash flows using the prevailing swap rate for the remaining time to maturity. The question now is how should one value the floating-rate part for which the future payments are not known in advance.

Let us study the value of a series of floating-rate cash inflows. This series of (as yet unknown) inflows must have the same market value as a short-term deposit where the principal amount is reinvested periodically. The reason for this equivalence is that the cash flows are the same, as the example will show. To buy such a series of deposits we need to buy only the currently outstanding deposit. No extra money is needed to redeposit the maturing principals later on.

## Example 7.7

Suppose that you want to replicate a risk-free USD 10,000 floating-rate bond with semiannual interest payments equal to the 6 -month T-bill rate, the first of which is due within four months. At the last reset date, the six-month T-bill rate was 3 percent p.a.; thus, the next interest payment equals $10,000 \times(1 / 2) \times 3 \%=$ USD 150. The current four-month rate of return is 0.9 percent (or 2.7 percent p.a., simple interest).

The above floating-rate bond can be replicated by "buying" uSD 10,150 due three months from now at a present value cost of USD $10 / 1.009=$ USD 10059.46. After four months, you withdraw USD 150 to replicate the bond's first coupon, and you redeposit the remaining 10,000 at the then prevailing six-month return. When this investment expires, you again withdraw the interest and redeposit the 10,000 at the then-prevailing rate, and you continue to do so until the bond expires. Notice that the future payoffs of the rolled-over deposit are identical to the payoffs of the floating rate bond. The cost to you was only the initial USD 10059.46. Then, by arbitrage, the floating rate bond is also worth 10059.46.

Figure 7.4: Crucial dates for valuing a floating-rate note


The general expression for the value of a floating-rate bond is

$$
\begin{equation*}
\text { Value of a risk-free floating-rate bond }=[\text { Face value }] \times \frac{1+r_{t_{0}, T_{1}}}{1+r_{t, T_{1}}}, \tag{7.8}
\end{equation*}
$$

where

$$
\begin{aligned}
t_{0} & \text { is the last reset date } \\
T_{1} & \text { is the next reset date } \\
t & \text { is the present date (the valuation date, with } \left.\mathrm{t}_{0}<\mathrm{t}<\mathrm{T}_{1}\right) \\
r_{t_{0}, T_{1}} & \text { is the coupon effectively payable at } T_{1} \\
r_{t, T_{1}} & \text { is the effective current return until time } T_{1} .
\end{aligned}
$$

The current market value of a coupon swap then equals the difference between the market value of the loan that underlies the incoming stream and the market value of the loan that underlies the outgoing stream.

## Example 7.8

Some time ago, a South-African company speculated on a drop in fixed-rate interest rates, and swapped ZAR 10 m at 7 percent, semi-annual and fixed, for ZAR 10 m at the six-month ZAR LIBOR. That is, the contract stipulates that, every six months, the firm pays the six-month ZAR LIBOR rate (divided by two) ${ }^{5}$ on a notional ZAR 10 m , and receives $7 / 2=3.5$ percent on the same notional amount. Suppose that ZAR medium-term interest rates have fallen substantially below 7 percent. The company reckons it has made a nice profit on its swap contract, and wants to lock in the gain by selling the swap. Current conditions are as follows:

- The swap has five years and two months left until maturity.
- The current five-year zar swap rate (for semiannual payments) is 5 percent p.a., meaning 2.5 percent every six months.
- The Zar libor rate, set four months ago for the current six-month period, is 4 percent p.a.

[^4]- The current two-month ZAR LIBOR is 3.5 percent p.a.

To value the (incoming) ZAR cash flows, note that there are eleven remaining interest payments at 3.5 percent each, the first of which is due two months from now. Discounted at $5 / 2=2.5$ percent per half-year, the value is: ${ }^{6}$

$$
\begin{align*}
\mathrm{PV}^{\text {fix }} & =10 m \times[1+(0.035-0.025) \times a(2.5 \%, 11)] \times 1.025^{4 / 6} \\
& =\text { ZAR } 11,133,193  \tag{7.9}\\
\mathrm{PV}^{\text {flo }} & =10 m \times \frac{1+1 / 2 \times 0.04}{1+2 / 12 \times 0.035}=\text { ZAR } 10,041,425 \tag{7.10}
\end{align*}
$$

So the value of the fixed-rate leg exceeds the value of the floating-rate leg by

$$
\begin{equation*}
\text { ZAR } 11,133,193-10,041,425=\operatorname{ZAR} 1,691768 \tag{7.11}
\end{equation*}
$$

This is what the company should receive for its swap contract.

### 7.3.2 Base Swaps

Under a base swap, the parties swap two streams of floating-rate interest payments where each stream is determined by a different base rate. For example, a LIBORbased revolving loan can be swapped for a us T-bill-based revolving loan. The spread between these two money-market rates is called the TED spread (treasuryeurodollar spread). The TED spread is non-zero because T-bills and euro-CDs are not perfect substitutes in terms of political risk $^{7}$ and default risk. TED swaps can be used either to speculate on changes in the TED spread, or to hedge a swap book containing contracts with different base rates.

## Example 7.9

The US office of a major bank has signed a fixed-for-floating swap based on the USD T-bill rate, while the London office has signed a floating-for-fixed swap based on USD LIBOR. This bank now has the USD T-bill rate as an income stream, and the USD LIBOR rate as an outgoing stream. To cover the TED-spread risk, it can swap its T-bill income stream for a LIBOR income stream using a base swap. The counterparty to this swap may be a speculator or simply another swap dealer who faces the opposite problem.

[^5]Figure 7.5: Renault's 1981 circus swap


### 7.4 Cross-Currency Swaps

The cross-currency swap, or circus swap, is a currency swap combined with an interest rate swap (floating versus fixed rate), in the sense that the loans on which the service schedules are based differ by currency and type of interest payment. An early example is the Renault-Yamaichi swap already mentioned in Section 7.2.1. The historic background for the swap is as follows:

- Renault, a French car producer, wanted to get rid of its USD floating rate debt, and wanted to borrow fixed-rate JPY instead. The snag was that, because of Japanese regulations at the time, Renault was not permitted to borrow in the Japanese market.
- Simultaneously, Yamaichi Securities was being encouraged by Japan's Ministry of Finance to buy USD assets. ${ }^{8}$ It could have bought, for instance, Renault's USD floating rate notes but was unwilling to take the exchange risk.
With the help of Bankers Trust, an investment bank, Renault convinced Yamaichi to borrow fixed-rate JPY and to buy floating-rate USD notes of similar rating and conditions as Renault's notes. As illustrated in Figure 7.5, Yamaichi was to hand over the USD service income from the USD investment to Renault, who would use the floating-rate USD interest stream to service its own floating-rate notes. As compensation, Renault undertook to service Yamaichi's equivalent fixed-rate yen loan, and pay a spread to both Bankers Trust and to Yamaichi.

The advantages of the swap to each party were:

- Renault was able to access the JPY capital market and get rid of its USD liability. (A more obvious solution would have been to borrow JPY and retire

[^6]the USD floating-rate notes with the proceeds. However, the first part of this transaction was not legally possible and the second part would have been expensive in terms of transaction costs or call premiums.)

- Yamaichi earned a commission. In addition, it now held USD assets (which was politically desirable) but these assets were fully hedged against exchange risk by the swap.
- Banker's Trust earned a commission on all of the payments that passed through its hands, plus a fee for arranging the deal.
The swap is also memorable because, even though it came quite soon after IBM-wB, it is already much more modern: it was not negotiated directly between two companies, but set up by Bankers Trust. Relatedly, there was no direct swap contract between Renault and Yamaichi, but two contracts (the double swap, as it was called then): Renault $v$ Bankers Trust, and Bankers Trust $v$ Yamaichi. This way, Bankers Trust took over the counterparty risk from Renault and Yamaichi, or, to be more precise, replaced the original counterparty risk by the risk that вт itself may get in trouble. ${ }^{9}$


### 7.5 CFO's Summary

The interest paid on any loan can be decomposed into the risk-free rate plus a spread that reflects the credit risk of the borrower. Swaps allow a company to borrow in the market where it can obtain the lowest spread, and then exchange the risk-free component of the loan's service payments for the risk-free component of another loan. This is useful if the other loan is preferred in terms of its currency of denomination or in terms of the way the periodic interest payments are determined (fixed or floating), as shown in Table 7.5. The use of risk-free rates within the swap is justified because the right of offset and the credit trigger eliminate virtually all risk from the swap - even though the company's ordinary loans remain risky. If desired, also the original loan's risk-spread payments can be swapped into the desired currency without altering their PV.

The difference between the spreads asked in different market segments usually reflects an information asymmetry-for instance, the firm's house bank often offers the best spreads because it is is less afraid of adverse selection - but may also reflect an interest subsidy. Another advantage is that the swap contract is a single contract, and is therefore likely to be cheaper than its replicating portfolio (borrowing in one market, converting the proceeds into another currency, and investing the resulting amount in another market). Other potential advantages include tax savings, or access to otherwise unavailable loans, or advantages of off-balance-sheet reporting.

[^7]Table 7.5: Swaps: Overview

|  |  |  | direct loan with best spread is ... |  |
| :--- | :--- | :---: | :---: | :---: |
| preferred loan is ... |  | fixed rate | floating rate |  |
| fixed rate | same currency | (do not swap) | do interest swap |  |
|  | other currency | fix/fix currency swap | do circus swap |  |
| floating rate | same currency | do interest swap | (do not swap) |  |
|  | other currency | do circus swap | flo/flo currency swap |  |

Depending on the combination of the preferred type of loan and the cheapest available loan, one could use a fixed-for-fixed swap, a fixed-for-floating swap, or a circus swap. Each of these swaps can also be used to speculate on changes in interest rates or exchange rates. Likewise, base swaps are used to hedge against, or to speculate on, changes in the TED spread.

These four swaps are just the most common types; in fact, many more exotic swap-like contracts are offered. Thus, swaps have become increasingly popular with financial institutions and large corporations. All of these swaps are based on the principle of initial equivalence of the two legs of the contract. Thus, like forward contracts on exchange rates or currencies, the initial value of a swap is zero. To compute the market value of such a contract after inception, we just value each leg in light of the prevailing exchange rates and interest rates.

### 7.6 TekNotes

## Technical Note 7.1 The value of a bond as a function of its yield to maturity

The valuation formula is derived as follows. Let the face value be 1 , the coupon $c$ per period, and the first coupon due exactly 1 period from now. (The yield is denoted by $R$, not $r$, because a compound per-period yield on a coupon bond should not be confused with an effective simple return on a zero-coupon bond.) We start with (almost) a definition; use the annuity formula; add and subtract 1 ; divide and multiply by $R$; and re-arrange:

$$
\begin{align*}
\mathrm{PV} & =\sum_{t=1}^{n} \frac{c}{(1+R)^{t}}+\frac{1}{(1+R)^{n}} \\
& =c \frac{1-(1+R)^{-n}}{R}+(1+R)^{-n} \\
& =c \frac{1-(1+R)^{-n}}{R}+(1+R)^{-n}-1+1 \\
& =c \frac{1-(1+R)^{-n}}{R}+R \frac{(1+R)^{-n}-1}{R}+1 \\
& =(c-R) \frac{1-(1+R)^{-n}}{R}+1=(c-R) a(n, R)+1 \tag{7.12}
\end{align*}
$$

If the time to the next coupon is $1-f$ rather than unity, the PV rises by a factor $(1+R)^{f}$.

### 7.7 Test Your Understanding

### 7.7.1 Quiz Questions

1. How does a fixed-for-fixed currency swap differ from a spot contract combined with a forward contract in the opposite direction?
2. Describe some predecessors to the currency swap, and discuss the differences with the modern swap contract.
3. What are the reasons why swaps may be useful for companies who want to borrow?
4. How are swaps valued in general? How does one value the floating-rate leg (if any), and why?

### 7.7.2 Applications

1. The modern long-term currency swap can be viewed as:
(a) a spot sale and a forward purchase.
(b) a combination of forward contracts, each of them having zero initial market value.
(c) a combination of forward contracts, each of them having, generally, a non-zero initial market value but with a zero initial market value for all of them taken together.
(d) a spot transaction and a combination of forward contracts, each of them having, generally, a non-zero initial market value but with a zero initial market value for all of them taken together.
2. The swap rate for a long-term swap is:
(a) the risk-free rate plus the spread usually paid by the borrower.
(b) the risk-free rate plus a spread that depends on the security offered on the loan.
(c) close to the risk-free rate, because the risk to the financial institution is very low.
(d) the average difference between the spot rate and forward rates for each of the maturities.
3. The general effect of a swap is:
(a) to replace the entire service payment schedule on a given loan by a new service payment schedule on an initially equivalent loan of another type (for instance, another currency, or another type of interest).
(b) to replace the risk-free component of the service payment schedule on a given loan by a risk-free component of the service payment schedule on an initially equivalent loan of another type (for instance, another currency, or another type of interest).
(c) to change the currency of a loan.
(d) to obtain a spot conversion at an attractive exchange rate.
4. You borrow USD 1 m for six months, and you lend EUR 1.5 m -an initially equivalent amount-for six months, at p.a. rates of 6 percent and 8 percent, respectively, with a right of offset. What is the equivalent spot and forward transaction?
5. Your firm has USD debt outstanding with a nominal value of USD 1 m and a coupon of 9 percent, payable annually. The first interest payment is due three months from now, and there are five more interest payments afterwards.
(a) If the yield at par on bonds with similar risk and time to maturity is 8 percent, what is the market value of this bond in USD? In Yen (at $S_{t}=$ JPY/USD 100)?
(b) Suppose that you want to exchange the service payments on this USD bond for the service payments of a 5.25-year JPY loan at the going yield, for this risk class, of 4 percent. What should be the terms of the JPY loan?
6. You borrow NOK 100 m at 10 percent for seven years, and you swap the loan into NZD at a spot rate of NOK/NZD 4 and the seven-year swap rates of 7 percent (NZD) and 8 percent (NOK). What are the payments on the loan, on the swap, and on the combination of them? Is there a gain if you could have borrowed NZD at 9 percent?
7. Use the same data as in the previous exercise, except that you now swap the loan into floating rate (at LIBOR). What are the payments on the loan, on the swap, and on the combination of them? Is there a gain if you could have borrowed EUR at LIBOR +1 percent?
8. You can borrow CAD at 8 percent, which is 2 percent above the swap rate, or at CAD LIBOR +1 percent. If you want to borrow at a fixed-rate, what is the best way: direct, or synthetic (that is, using a floating-rate loan and a swap)?
9. You have an outstanding fixed-for-fixed NOK/NZD swap for NOK 100 m , based on a historic spot rate of NOK/NZD 4 and initial seven-year swap rates of 7 percent (NZD) and 8 percent (NOK). The swap now has three years to go, and the current rates at NOK/NZD 4.5, 6 percent (NZD three years), and 5 percent (nok three years). What is the market value of the swap contract?
10. Use the same data as in the previous exercise, except that now the NZD leg is a floating rate. The rate has just been reset. What is the market value of the swap?

[^0]:    ${ }^{1}$ Critical economist would rightly object that this is a saving only to the extent the difference in the risk spreads was irrational.

[^1]:    ${ }^{2}$ See TekNote 7.1 if the formula is new to you.

[^2]:    ${ }^{3}$ The N -year yield at par is the coupon that has to be assigned to an N -year bond in order to give it a market value equal to the par value (the principal). If the parties want a cash flow pattern that differs from the single-amortization ("bullet") loan, the swap dealer is usually willing to design a contract that deviates from the standard form, but at a different swap rate. See the subsection on non-bullet loans, further in this section.

[^3]:    ${ }^{4}$ Note that, since these two amounts are in different currencies with a stochastic future exchange rate, there is no way to amalgamate them into one number or one percentage. That is, 3 percent in USD and 0.4 percent in JPY is not 3.4 percent.

[^4]:    ${ }^{5}$ This linear annualisation/deannualisation when interest is due more then once a year is the convention for bonds, notes and loans. It would have been more correct to annualise the effective rate, $2.5 \%$ per semester, via compound interest into $1.025^{2}-1=5.0625 \%$ because we use compounding in the valuation too. But life is full of inconsistencies.

[^5]:    ${ }^{6}$ The correction $1.025^{4 / 6}$ reflects the fact that the last coupon was four months ago; that is, we value the bond $4 / 6$ ths into the first coming coupon period, not at the beginning of that period.
    ${ }^{7}$ Dollar deposits in London cannot be blocked by the US Government, which is attractive to some parties. This is not a major issue anymore.

[^6]:    ${ }^{8}$ Japan wanted to show it was doing its bit to help finance the us deficit and also help "recycle the Petrodollars", a big issue after the second oil shock, early eighties.

[^7]:    ${ }^{9}$ BT did get in trouble, and was taken over by Deutsche bank. Also Yamaichi sank ignominiously and was absorbed by Nomura, Japan's largest broker and investment bank.

